Obtaining Corrosion Rates by Bayesian Estimation: Numerical Simulation Coupled with Data

by Kenji Amaya, Naoki Yoneya, and Yuki Onishi

Protecting structures from corrosion is one of the most important challenges in engineering. Cathodic protection using sacrificial anodes or impressing current from electrodes is applied to many marine structures. Prediction of the corrosion rates of structures and the design of cathodic protection systems have been traditionally based on past experience with a limited number of empirical formulae.

Recently, application of numerical methods such as the boundary element method (BEM) or finite element method (FEM) to corrosion problems has been studied intensively,[1-8] and these methods have become powerful tools in the study of corrosion problems.[23-24]

With the progress in numerical simulations, "Inverse Problems" have received a great deal of attention. The "Inverse Problem" is a research methodology pertaining to identifying unknown information from external or indirect observation utilizing a model of the system, as shown in Fig. 1.

The background of inverse problems is the modeling and simulation of natural phenomena. When observations are taken of these phenomena, the observation data are used to infer knowledge about physical states. One of the most spectacular successes in the field of inverse problems was the invention of an inversion algorithm for computed tomography by Cormack (1963) and its experimental demonstration by Hounsfield (1973). The two shared the Nobel Prize in Physiology/Medicine in 1979. Applications of inverse problems arise in many fields of engineering as well. In the field of corrosion engineering, there are many issues that benefit from an inverse analysis approach.

In this article, the use of numerical simulation for evaluating corrosion rates is explained with the boundary element method as an example. Then, an inverse analysis method for identifying corrosion rates or cathodic protection currents from the (easily measured) potential distribution around marine structures is introduced. This method is based on the Bayesian estimation, with the measured data and numerical simulation focused on the potential distributions around a seaside structure.

Electrochemical Aspects and Mathematical Model

We consider a typical metal M in an electrolyte. Both anodic and cathodic reactions occur simultaneously on the metal surface according to

$$\text{Anodic reaction: } M \rightarrow M^{n+}\ ne^{-} \tag{1}$$

$$\text{Cathodic reaction: } \frac{1}{2}O_2 + H_2O + 2e^{-} \rightarrow 2OH^{-} \tag{2}$$

Due to these reactions, an electric current flows. The density of the electric current across the metal surface (i) versus electrical potential in the electrolyte near the metal surface against a reference electrode, *e.g.*, saturated calomel electrode (SCE) – (E) curve is called a polarization curve, which is schematically shown in Fig. 2. The relationship between the current density (i) and the potential E for anodic and cathodic reactions (solid curves) are not obtained individually; only the nominal relationship $E = f(i)$ for the two reactions (dashed curve) are measured.

In the natural state, the reactions become balanced at point C in Fig. 2, and the current corresponding to CD flows from the anode to the cathode. The corrosion rate is proportional to this anodic current density CD. It is possible to suppress the anodic current density by impressing a current onto the metal through the electrolyte using an external power supply, or by connecting the metal object with a more base metal, *i.e.*, sacrificial anode, and reducing the potential of the metal to the critical value E_p. This method is called cathodic protection (CP).[18]

We assume that the surface of the electrolyte domain Ω is surrounded by $\Gamma(=\Gamma_d + \Gamma_n + \Gamma_m)$ as shown in Fig. 3, where Γ_m is the metal surface, and the potential and current densities are prescribed on Γ_d and Γ_n, respectively. The potential field in the homogeneous electrolyte can be modeled mathematically by the Laplace's Eq. 3 under boundary conditions Eq. 4 through Eq. 6 [8]

$$\nabla^2\phi = 0 \tag{3}$$

$$\phi = \phi_0 \quad \text{on } \Gamma_d \tag{4}$$

$$i\ (\equiv \kappa\frac{\partial\phi}{\partial n}) = i_0 \quad \text{on } \Gamma_n \tag{5}$$

$$-\phi = f(i) \quad \text{on } \Gamma_m \tag{6}$$

where κ denotes the conductivity of the electrolyte, and $\partial/\partial n$ the outward normal derivative.

Note that the potential ϕ is defined with reference to the metal and has an inverse sign to that usually employed in corrosion problems, wherein the potential is defined against a reference electrode such as SCE. ϕ_0 and i_0 are the prescribed values of the potential and the current density, respectively. The function $f(i)$ is the experimentally determined polarization curve as stated above. In case the corroding structure consists of multiple materials, the number of polarization curves is the same as that of the material types.

By solving Eq. 3 under the boundary conditions Eq. 4-6, the potential near the metal surface and the current density that is proportional to the corrosion rate can be determined. Because the knowledge of physical quantities on the metal surfaces is important, a boundary element method in which only the surface of analytical domains must be discretized with elements is employed here. The standard boundary element procedures[19-20] lead to

$$\kappa[H]\begin{Bmatrix}\phi_0\\ \phi_n\\ -f(i_m)\end{Bmatrix} - [G]\begin{Bmatrix} i_d\\ i_0\\ i_m\end{Bmatrix} = 0 \tag{7}$$

where the detailed expressions of matrices, $[H]$ and $[G]$, are given in References 19 and 20 and the subscripts d, n, and m represent the quantities on Γ_d, Γ_n, and Γ_m, respectively. The system of nonlinear algebraic equations, Eq. 7, is solved by using an iterative procedure, *e.g.*, the Newton-Raphson method.[21] An experimental verification of the boundary element solution is shown in References 8 through 22.

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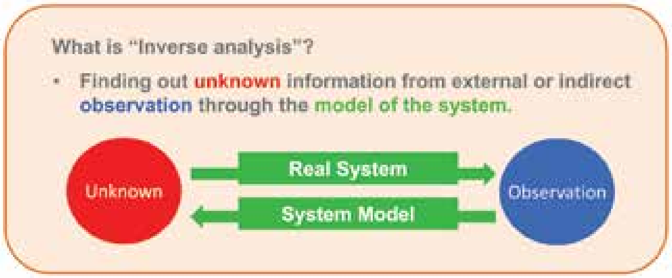

Fig. 1. *Concept of inverse problems.*

Inverse Problem Approach

As described above, the corrosion rate of a structure can be estimated by solving Laplace's equation with nonlinear boundary conditions, which are based on experimentally determined electrochemical polarization curves. However, the electrochemical polarization curves of the materials considered are not available in most practical cases. In such a case, the inverse approach, in which the current density on the surface of a metal structure is estimated from the measured potential in the electrolyte, is necessary. This type of inverse problem will be discussed here.

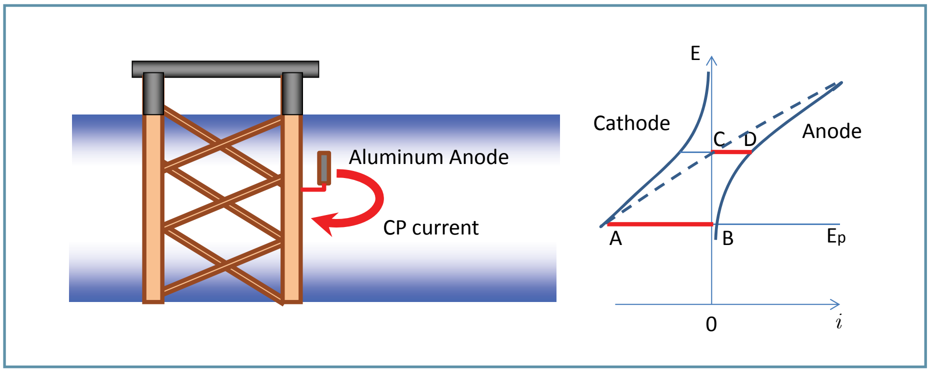

Fig. 2. *Schematic view of polarization curve.*

As shown in Fig. 4, the boundary and supplementary conditions to be treated in the inverse problem are given by

$$\phi = \phi_0 \text{ on } \Gamma_d \tag{8}$$

$$i\,(\equiv \kappa \frac{\partial \phi}{\partial n}) = i_0 \text{ on } \Gamma_n \tag{9}$$

$$\phi = \bar{\phi} \text{ on } \Gamma_s \tag{10}$$

In the inverse analysis, the boundary conditions of domain Ω are not fully given, but the value of potential can be obtained at several points in the electrolyte by practical measurement to compensate for this lack of boundary conditions. However, even if the number of the measured potential datapoints is increased sufficiently to permit solving the inverse problem, the solution would be abnormally unstable due to the ill-posed problem.[25]

Hence, we make use of the *a priori* information that ϕ and i on Γ_m must satisfy the polarization characteristics.[26] At first, we approximate the polarization curve by a function, *e.g.*, the Tafel expression[27]

$$-\phi = f_m\,(i;\alpha_j) \text{ on } \Gamma_m \tag{11}$$

where α_j (j = 1, 2, ..., L; L = total number of parameters) is a parameter in the function. Then, the many parameters in the function are estimated by an inverse analysis. After obtaining the polarization characteristics, the potential distribution is easily calculated by direct analysis.

Following the usual boundary element formulation with given boundary conditions, we obtain

$$\kappa[H]\begin{Bmatrix} \phi_0 \\ \phi \\ -f(i;\,\alpha_j) \end{Bmatrix} - [G]\begin{Bmatrix} i \\ i_0 \\ i \end{Bmatrix} = 0 \tag{12}$$

By considering a set of initial values of the parameters α_j as the unknown or latent variables in the inverse problem, the following observation equation can be calculated by solving the direct problem Eq. 12

$$\phi_s = h(\alpha_j) + v \tag{13}$$

where ϕ_s is the vector containing the value of potential at measurement points, and $h(\ )$ is the vector model function which represents the relationship between α_j and ϕ_s. Also, v is a random variable vector corresponding to measurement noise and model error in the potential.

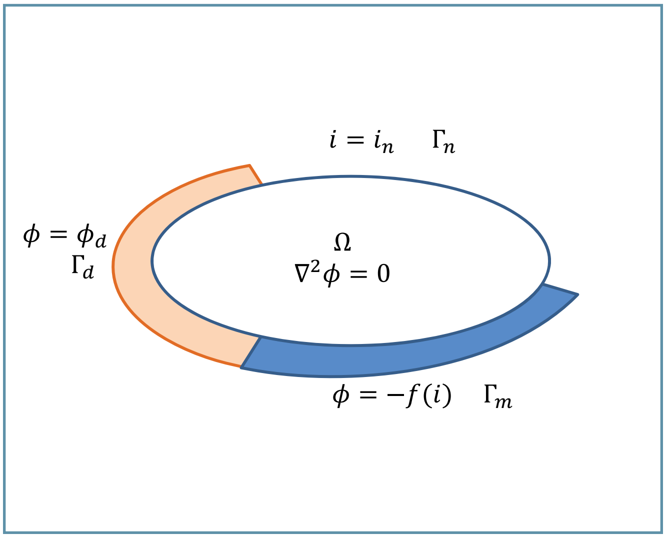

Fig. 3. *Governing equation and boundary conditions.*

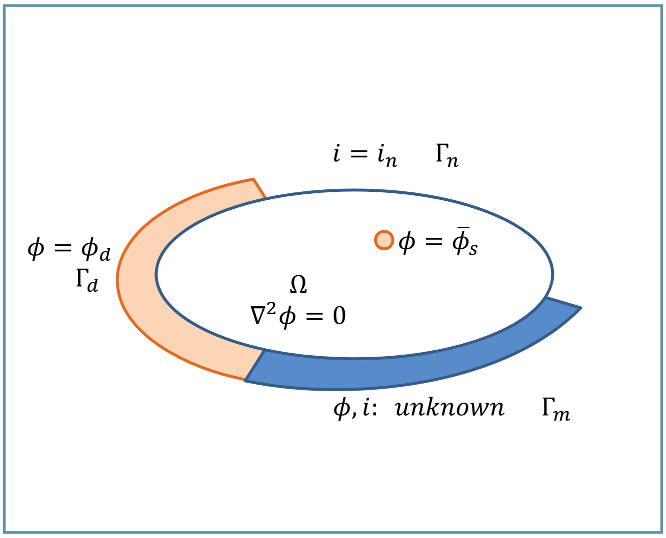

Fig. 4. *Governing equation and boundary conditions for inverse problem.*

The inverse problem identifying the corrosion rate can be formulated as a maximum likelihood estimation (MLE) by regarding α_j as random variables, whose solution can be obtained by

$$\hat{\alpha}_j = \underset{\alpha_j}{\arg\min}\{-\ln p\,(\bar{\phi}_s \mid \alpha_j)\} \tag{14}$$

$$= \underset{\alpha_j}{\arg\min}\{-\ln p_v\,(\bar{\phi}_s - h(\alpha_j))\} \tag{15}$$

where, $p_v(\,)$ is the probability density function of v, and $\bar{\phi}$ is the vector containing the measured potential data. This optimization calculation is repeated by modifying the parameters employing a minimizing technique, *e.g.*, the conjugate gradient method. Even if the above procedure is followed, the solution for α_j is sometimes unstable and it depends on the assumed initial values. In such a case, the Bayesian Estimation approach, which utilizes *a priori* information, is sometimes effective. For example, the *a priori* information could be that if the metal structure of interest is made of a low alloy steel, then the parameters of the polarization characteristics curve α_j must be within some range. This kind of information is often available in engineering problems. Such information is easily expressed in the form of fuzzy membership functions $p_m(\alpha_j)$.[27]

The Bayesian Estimation solution can be formulated as

$$\hat{\alpha}_j = \underset{\alpha_j}{\arg\min}\{-\ln p\,(\bar{\phi}_s \mid \alpha_j) - \ln p_m(a_j)\} \tag{16}$$

$$= \underset{\alpha_j}{\arg\min}\{-\ln p_v\,(\bar{\phi}_s - h(\alpha_j) - \ln p_m(a_j)\} \tag{17}$$

The above approach is also termed "maximum *a posteriori*" (MAP) estimation. The mean value of the likelihood function can be the solution as well.

Verification

Our primary aim is to verify the applicability of the inverse analysis in the estimation of the polarization characteristics. To concentrate on the evaluation of the process, we have created necessary measured data $\bar{\phi}_s$ through a regular/direct boundary element analysis instead of carrying out real potential measurements. Such a procedure allows the present technique to be isolated from other factors that can influence the accuracy and the convergence rate. However, to accommodate an important aspect of experimental measurements, a small perturbation/error is added to the calculated results. These values are assumed to be the measured potential value $\bar{\phi}_s$ and used in the inverse analysis to determine the polarization characteristics.

Direct analysis.—Let us consider a ship that has six electrodes and six sensors as shown in Fig. 5. At first, we perform a direct analysis for the case where the current densities impressed to the electrodes on the left side of the hull are 0.04, 0.04, and 0.4 [A/m^2] (from head to tail), and those on the right side of the hull are 0.01, 0.01, and 0.01 [A/m^2] (from head to tail). We assume that that the hull is made of painted low alloy steel plates and their polarization characteristics are given by

$$\phi = -0.600\ \sinh^{-1}(200i) + 0.650 \tag{18}$$

where the units of ϕ and i are [V] and [A/m^2], respectively.

The calculation is performed by employing 764 constant elements. The estimated potential distribution is shown in Fig. 6.

The potentials at the location of the sensors are rounded off to three top figures to take into account the measurement accuracy, and are used as the input data in the following inverse analysis.

Estimation of potential distribution.—In this inverse problem, we assume that only the potential data at the location of the sensors is available. At first, we estimate the polarization characteristics. We assume that the polarization characteristics of the painted hull are represented in the following form

$$\phi = -\alpha_1\ \sinh^{-1}(\alpha_2 i) + \alpha_3 \tag{19}$$

where $\alpha_j (j = 1, 2, 3)$ is the parameter to be estimated.

We use *a priori* information about polarization characteristics, the membership functions of which are shown in Fig. 7. After estimating these parameters, we calculate the potential distribution by a direct analysis using the estimated polarization characteristics. The estimated potential distribution is shown in Fig. 8 and is found to agree well with the exact solution (Fig. 6).

Practical application.—The inverse analysis method is applied for a real truss structure. The overview of the truss type structure is shown in Fig. 9. Over 90 pieces of aluminum anodes are attached to this structure. First, the actual potential distribution around the structure

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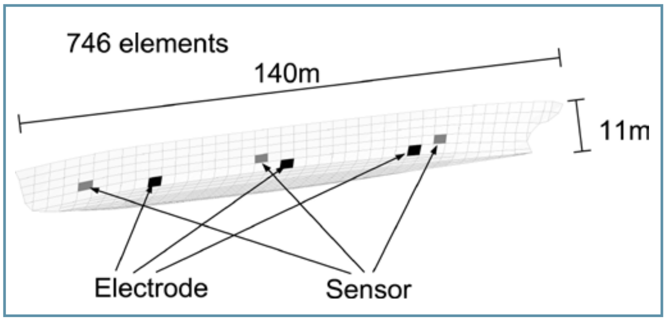

Fig. 5. *Location of electrodes and sensors.*

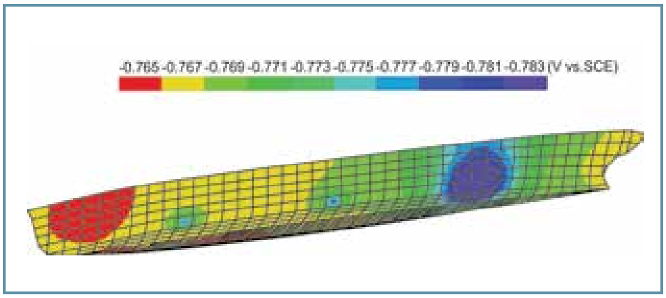

Fig. 6. *Potential distribution obtained by a direct analysis.*

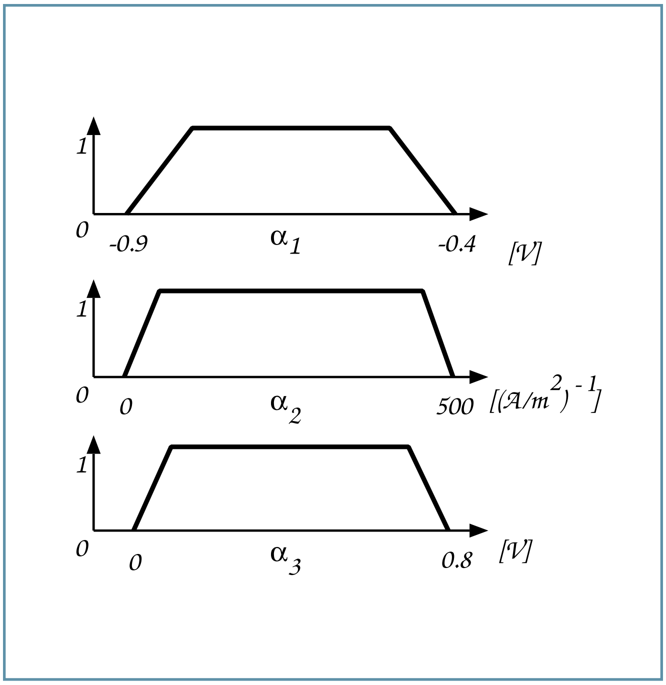

Fig. 7. A-priori *information of polarization characteristics as membership functions.*

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is measured as shown in Fig. 9. Then, the relationship between the CP current and the potential distribution, which is represented by the observation equation, is quantitatively examined. A 3D finite element method is employed to simulate the potential distribution. Finally, Bayesian estimation is performed to estimate the CP current. The estimated potential distribution is shown in Fig. 10. It could be confirmed that the Bayesian estimation worked well for these kinds of corrosion engineering problems.

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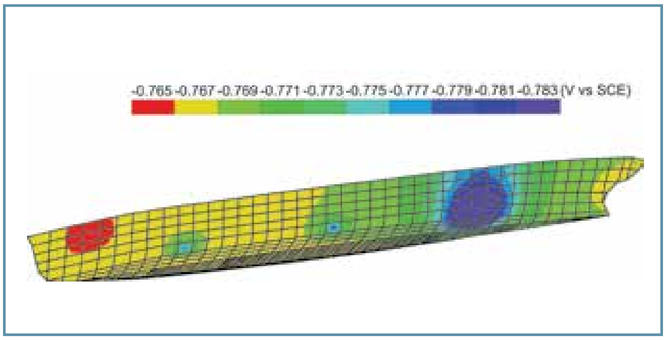

Fig. 8. *Estimated potential distribution.*

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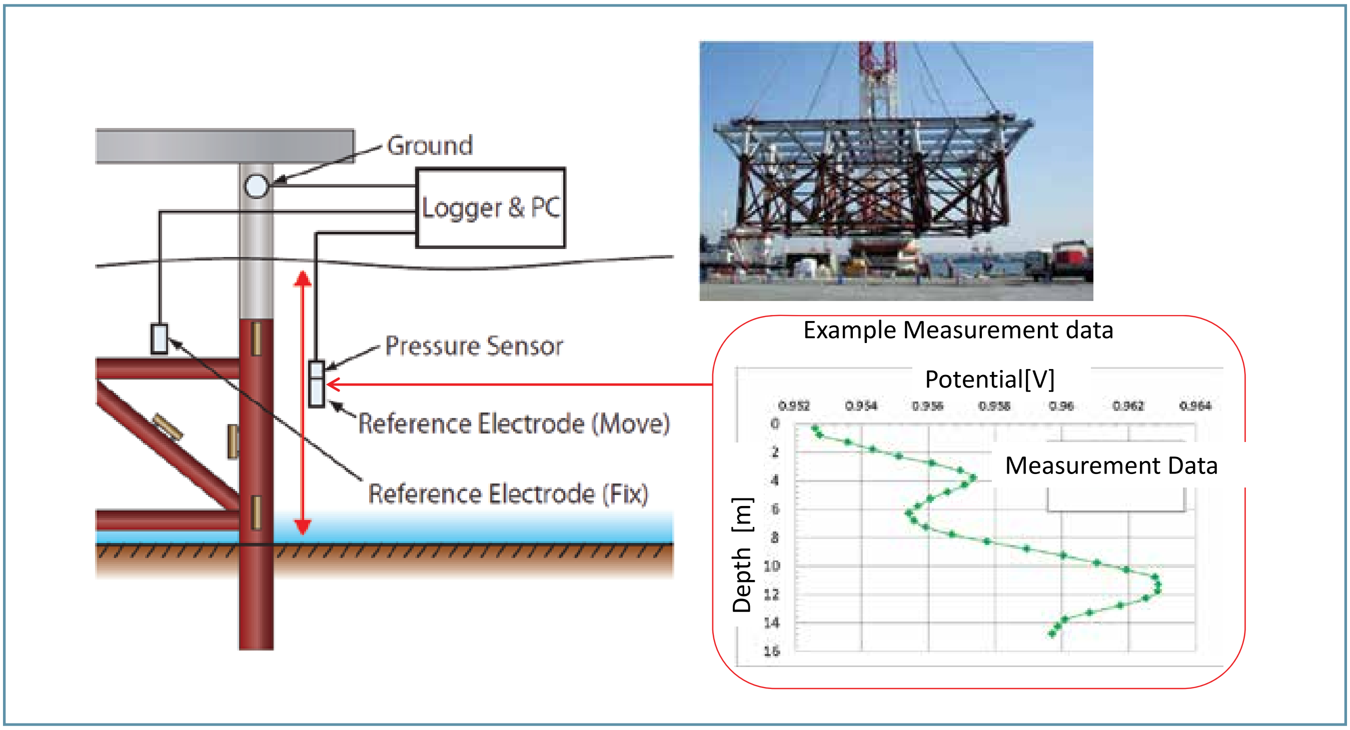

Fig. 9. *Measured potential distribution around the truss structure in Tokyo Bay.*

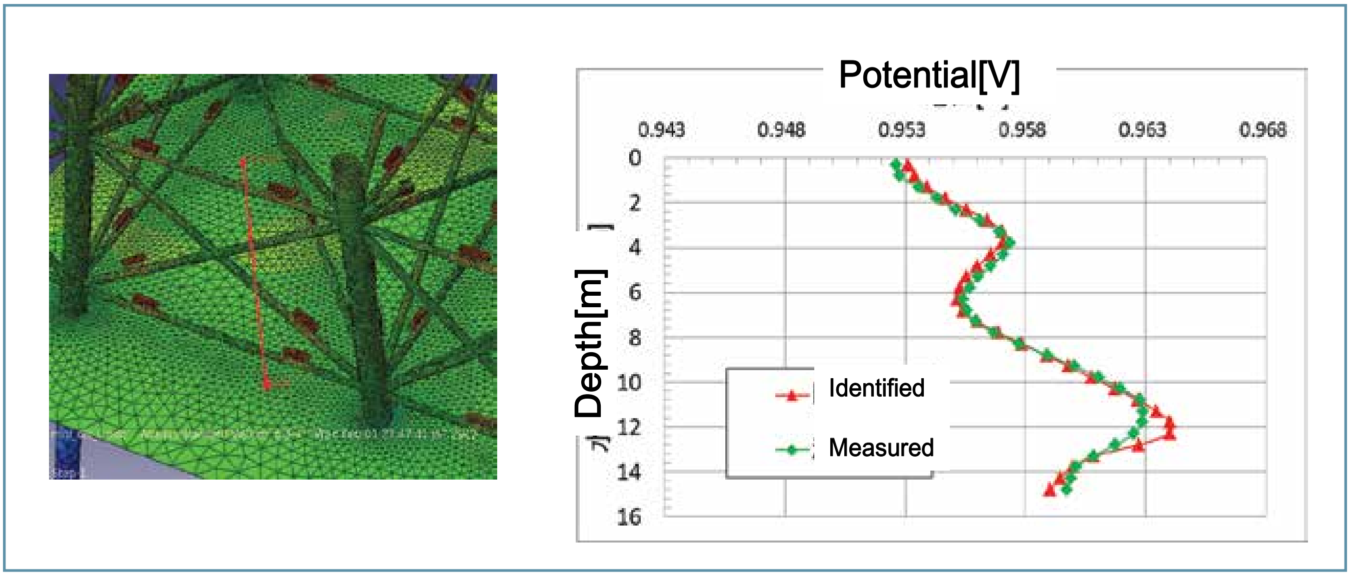

Fig. 10. *Identified potential distribution of whole surface of the truss structure in Tokyo Bay.*

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