Peltier Effect Related Correction to Ohmic Resistance

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It is well known that ohmic measurements are carried out at low current density in order to prevent heating. Usually, only the Joule heat is considered to be important. In contrast to the Joule heat, the Peltier and Thomson effects are linear in the current. As was shown in [1,2] the Peltier effect which is linear in current results in a correction to the resistance measured. Under current carrying conditions one of the sample contacts is heated and the other is cooled because of Peltier effect. The temperature gradient is proportional to the current. The Thomson heat is then proportional to square of the current and can therefore be neglected. Finally, the voltage swing across the circuit includes the thermoelectromotive force induced by the Peltier effect, which is linear in current. Accordingly, there exists a thermal correction to the ohmic resistance of the sample. The correction should be in comparison with ohmic resistance of the conductor. Above some critical frequency dependent on thermal inertial effects the correction disappears.

We first consider an isotropic (or of cubic symmetry) conductor that can be in the thermodynamic non-equilibrium with respect to the conducting electrons. In general, the current density $\boldsymbol{j}$ and the energy flux density $\boldsymbol{q}$ of the inhomogeneous conductor are given by

$$\vec{j} = \sigma(\vec{E} - \alpha\nabla T), \quad \vec{q} = (\phi + \alpha T)\vec{j} - \kappa\nabla T, \qquad (1)$$

where σ is the conductivity, α is the thermopower, κ is the thermal conductivity, $\phi = \varphi - \mu/e$ is electrochemical potential. For the steady state, div$\boldsymbol{j}$=0, then

$$Q = -div\,\vec{q} = div\,\kappa\nabla T + j^2/\sigma - jT\nabla\alpha = 0, \qquad (2)$$

where Q is the total amount of heat evolved per unit time and volume of the conductor. The current is accompanied by both the Joule and Thomson heats that are proportional to the second (first) power of the current respectively.

We now consider the thermal effects in connection with ohmic measurements of the conductor resistance. The conductor is connected by means of two identical extra leads to the current source. Both contacts assumed to be ohmic, α, σ, κ, the length l, and the cross-section S are different for the leads and the sample. The voltage is measured between the open ends ("c" and "d") kept at the temperature T_0 of the external thermal reservoir. In general, the contacts ("a" and "b") could be at different respective temperatures T_a and T_b.

It is well known that Peltier heat is generated by the current crossing the contact of two different conductors. At the contact (let us say "a"), the temperature T_a, the electrochemical potential ϕ, the normal components of the current $I=jS$, and the total energy flux qS are continuous. There exists the difference of thermopowers $\Delta\alpha = \alpha_1 - \alpha_2$. For $\Delta\alpha > 0$, the charge intersecting contact "a" gains the energy $e\Delta\alpha T_a$. Consequently, $Q_a = I\Delta\alpha T_a$ is the amount of the Peltier heat evolved per unit time in contact "a". We stress that Q_a can be calculated directly through the Thomson term in Eq.(2) as $Q_a = -\int IT\nabla\alpha dx$, where the integration is taken over the contact length. In fact, the Peltier effect is equivalent to the Thomson effect established at the contact.

For $\Delta\alpha > 0$ and the current flowing initially through the contact "a" the latter is heated. Then, the another sample contact "b" is cooled. The contacts are at different temperatures and $T_a - T_b = \Delta T > 0$. Using Eq. (1), we find the voltage swing U between ends "c" and "d" as

$$U = \int_c^d (j/\sigma + \alpha\nabla T)dx = RI + \varepsilon_T, \qquad (3)$$

where R is the total resistance of the circuit. The first term in Eq. (3) corresponds to the Ohm's law. The second term coincides with the expression for the conventional thermoelectromotive force under zero current conditions[3]. We notice that $\varepsilon_T = \int_c^d \alpha dT$ is a universal value because it only depends on the contact temperatures for arbitrary cooling conditions. There exists a correlation between the thermoelectromotive force and the Peltier and Thomson heats. Indeed, the total power evolved in the circuit UI is the sum of the Joule heat RI^2 and related to the thermal effects the power $\varepsilon_T I$. The product $\varepsilon_T I$ is then exactly the sum of the Peltier heat $Q_P = I\Delta\alpha\Delta T$ evolved at both contacts and the Thomson heat $Q_T = \int_c^d -IT\nabla\alpha dx$ in the conductor bulk. We conclude that for an arbitrary circuit under the same contact temperatures (T_a, T_b, and T_0) the zero current measurements of the thermoelectromotive force allow one to find the total amount of both the Peltier and Thomson heats at $I \to 0$.

We recall that the sample contacts are always extra heated (or cooled) because of the Peltier effect. The difference of the contact temperatures ΔT is linear in current, and therefore, there exists a thermal correction to the ohmic resistance $\Delta R = \varepsilon_T/I = U/I - R$. For simplicity, we assume σ, α, and κ are temperature independent. The thermoelectromotive force is then given by $\varepsilon_T = \Delta\alpha\Delta T$. Using Eqs. (2,3), one can find the voltage swing U and, thus, the thermal correction ΔR for an arbitrary circuit. We emphasize that the real cooling conditions strongly influence ΔR. For simplicity, we consider the adiabatic conditions with the sample being thermally isolated from the environment, then neglect the heat transfer within the leads. We emphasize that under the above conditions, the sample is not heated. In fact, at small current $T_a \approx T_a \approx T_0$ and hence, the amount Peltier heat evolved at contact "a" is equal to the one absorbed at contact "b". Recalling that the energy flux qS is continuous at each contact, from Eq. (3) we find the thermal correction to resistivity as

$$\Delta\rho = \frac{T_0(\Delta\alpha)^2}{\kappa} \qquad (4)$$

According to Eq.(4), $\Delta\rho$ depends on the reservoir temperature and the heat conductivity of the sample. We emphasize that the thermal correction is always positive, because the total amount of the Peltier heat $Q_P = \Delta RI^2 > 0$.

We now estimate the magnitude of the thermal correction to resistivity in the case where both the conductor and leads are metals. At room temperature, the electron heat conductivity and thermopower are given by $\kappa = L\sigma T$ and $\alpha = \pi^2 k\zeta/2e$, where $L = (\pi k/e)^2/3$ is the Lorentz number and $\zeta = kT/\mu$ is the degeneracy parameter. Hence, we find that $\Delta\rho/\rho \sim \zeta^2 << 1$. The thermal correction is small compared with the ohmic resistance because the electron gas is degenerate. In contrast, for semimetals(bismuth, $\mu \approx 35$meV) or non-degenerated semiconductors($\alpha \approx k/e$), the thermal correction can be comparable with the ohmic resistivity. We stress that $\Delta\rho$ is smaller under realistic cooling conditions. Moreover, above some critical frequency dependent on thermal inertial effects the thermal correction disappears.

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