Equilibrium Theory of Nanotips

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We model the microscopic deformation of a cathode surface in an intense electric field. Such mechanical side effects can be expected in Spindt tips, since Maxwell stresses of order $p = E^2/2\epsilon_0 = 1100$atm will be typical in the nanometer-scale region where electric fields are typically of order $E = 5$V/nm. Even for the lower fields used in "seasoning" a tip, surface diffusion of metal atoms may allow the surface near the apex to gradually evolve toward an equilibrium nanostructure. Here we report an analytic expression for the equilibrium nano-geometric response to macroscopic controls such as anode voltage.

We approximate the unperturbed surface as locally planar, we neglect electron emission, and we exclude very intense electric fields so that we can linearize the response of the surface. Consider the cylindrical region $r \leq R$ as shown in Table I between a grounded metallic cathode $z \leq 0$ and a parallel anode $z = d \gg R$. Any deviation $z = h(r)$ of the cathode surface changes the boundary condition and therefore v and $\vec{E} = -\nabla v(r, z)$. The equilibrium (h, v) state can be found by minimizing the free energy functional[Ljepojevic and Forbes, Proc. R. Soc. Lond. A (1955),Eq. 22]

$$\Psi = F_0 + \int_A (\gamma - \frac{1}{2}\sigma V) dA \tag{1}$$

where A is the perturbed surface, γ is its surface tension, σ is its surface charge density, and F_0 can be neglected.

We assume that the perturbations are local $h(r > R) = 0$, that they keep the volume of metal constant $\int_0^R h(r) r dr = 0$, and that the perturbed surface is smooth so that $h'(r = R) = 0$.

In the vacuum, the potential obeys Laplace's equation $\nabla^2 v(r, z) = 0$ and it vanishes on the perturbed surface $v(r, z = h(r)) = 0$ since the cathode is grounded. Any solution for this electrostatic problem can be written

$$v(r, z) = -zE_1 + \sum_n a_n J_0(k_n r) e^{-k_n z} \tag{2}$$

where $E_1 = -V/d$ is the unperturbed electric field, the a_n are adjustable parameters, and each k_n is defined in terms of the n^{th} zero of the cylindrical Bessel function J_0 of order zero, so that $J_0(k_n R) = 0$. Any symmetric deformation of the surface can be written in terms of the same Bessel functions

$$h(r) = \sum_n b_n J_0(k_n r). \tag{3}$$

We approximate the perturbation with the first few a_n and b_n. If there are N of each, then we begin with $2N$ degrees of freedom. We use N of these to satisfy the condition $v(r, z = h(r)) = 0$ another one for smoothness, one more for constant volume, and a final parameter, say a_1, to minimize Eq. 1. In this case $N = 3$ and we calculate that

$$b_n = \frac{a_n}{E_1}, \quad a_2 = 5.449\ a_1, \text{ and } \quad a_3 = 3.825\ a_1, \tag{4}$$

which allows us to write any self-consistent Ψ in terms of a_1. The minimum of Ψ determines the equilibrium height of the nanotip

$$h_0 = h(0) = .0163 \frac{\epsilon_0 V |E_1| R}{\gamma} \tag{5}$$

in SI units. For a lithium cathode with $\gamma = 0.3$ J/m^2, and for $V = 1$kiloVolt and $E_1 = -1 \times 10^9$V/m, we have $h_0 = .481R$, using $\epsilon_0 = 8.85 \times 10^{12}$F/m. Typical shapes are shown in Table I.

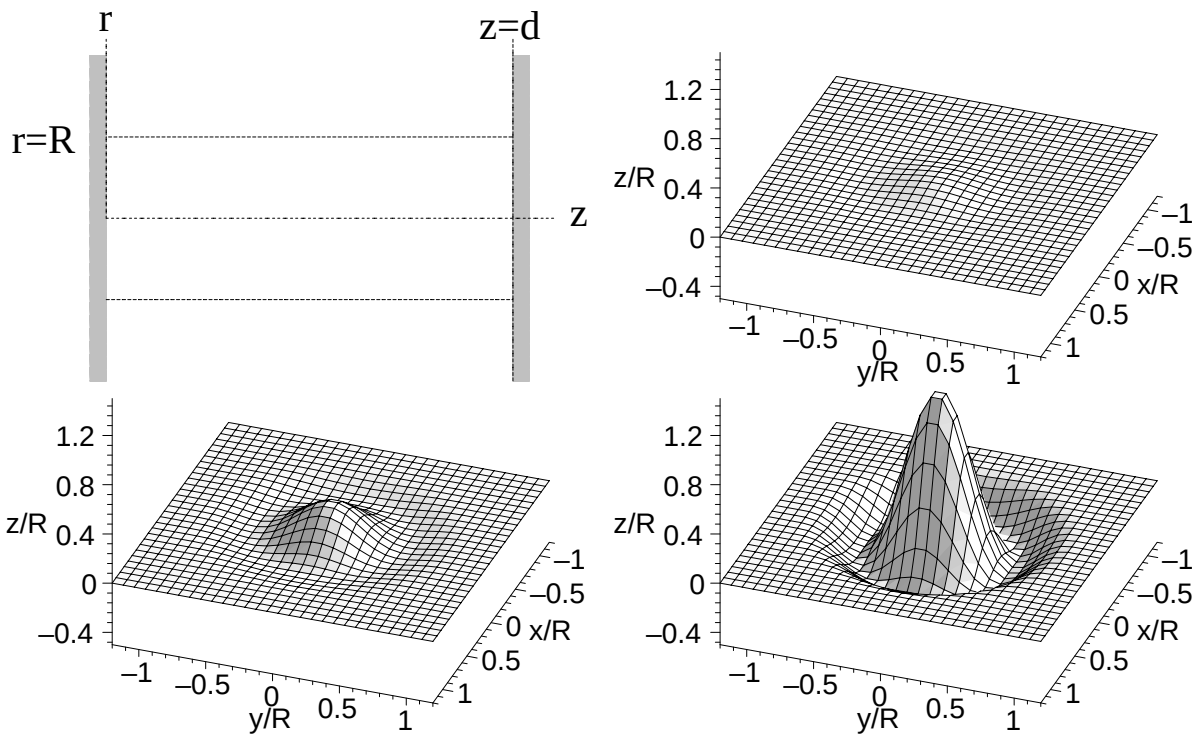
We can predict several observable effects from Eq. 5. Any electric field E_1 that can form a nanotip will be enhanced near the apex of the nanotip by the nanometric aspect ratio $\beta_{nano} \approx h_0/R$ if it is not shielded by adjacent nanotips. The nanoscale-enhanced field $E_{nano} = \beta_{nano} E_1 \propto V E_1^2$ increases faster than the local field E_1 and exhibits a non-geometric direct dependence on the anode voltage V. Nano-enhancement could further extend the nanotip, reduce the threshold voltage for field emission, increase the emission current, or all of these.

This result applies immediately to Spindt basetips. Just below the threshold of electron emission, we predict that one or more nanotips will grow near the apex of a Spindt tip. In this case, R is the radius of the apex of the basetip and the local electric field $E_1 = -\beta V$ begins with the conventional relation between anode voltage V and the effective electric field E_1 required for Fowler-Nordheim fitting. In this case, Eq. 5 becomes $h_0 = .0163 \frac{\epsilon_0 \beta V^2 R}{\gamma}$ so that emission from the bare Spindt tip should be surpassed by nanotip emission which will obey the Fowler-Nordheim expression but with an effective field of the form $E_{nano} \propto \beta V^3$, according to the proposed theory.

In conclusion, we find that the electric field required for Fowler-Nordheim emission can modify the geometry of a metallic field emitter. We expect one or more nanotips with aspect ratios of $h_0/R \approx \frac{\epsilon_0 \beta V^2}{60\gamma}$ to form near the apex of any field emitter.

Table I

UPPER LEFT: Cylindrical coordinate system. the shape of lithium nanotip for $V = 1$kV and three local field strengths: UPPER RIGHT: $200V/\mu$m, LOWER LEFT: 1 V/nm, and LOWER RIGHT: 5 V/nm.



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